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Suppression of crack growth in hybrid fibre composites

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A theoretical analysis based on the assumed form of the strain field surrounding a crack bridged by reinforcing elements has been used to examine the growth of a crack propagating transversely to the fibres in hybrid fibre composites. An intermingled carbon fibre/glass fibre polymer matrix system has been considered. Two situations have been investigated. In the first of these the effect of the addition of carbon fibres on the development of cracks resulting from the failure of the glass fibres by stress corrosion has been studied. The analysis indicates that crack growth can be severely inhibited by a 5% volume fraction of type III carbon fibres. The analysis has been used also to investigate the process by which strong high failing strain glass fibres inhibit the growth of cracks caused by the fracture of localized clusters of low failing strain carbon fibres. The predictions of this analysis agree with existing experimental data on glass fibre/carbon fibre hybrids.

1. Introduction

An analysis of the mechanics of the growth of a matrix crack bridged orthogonally by reinforcing members has previously been proposed [1]. This analysis has been based on the assumed form of the strain field surrounding the matrix crack from which the rates of release and absorption of energy with increasing crack length can be calculated. Following Griffith [2], unstable crack growth is assumed to occur when the rate of release of strain energy with increasing crack length exceeds the rate of absorption of energy. This analytical procedure was found to predict with reasonable accuracy [3], the conditions for the unstable growth of a crack in a metal sheet bridged by reinforcing members. The model was subsequently modified to deal with the growth of transverse cracks in laminates [4], and with the effects of non-uniform fibre strengths on crack stability [5]. Further work has shown the model to predict the so-called "hole size effect" on the strength of laminates and also to predict successfully the tensile strength of unidirectionally reinforced composites [6]. More recently, direct experimental observations of the strain field surrounding a crack bridged by reinforcing members have been shown to be in reasonable agreement with the assumed form of the strain field used in the model [7].

In this paper the analysis is used to predict the stability of cracks in intermingled hybrid fibre reinforced systems. Two sets of fibres are assumed to be present one set failing whilst the other remains intact bridging a crack of arbitrary length. The reinforcement is assumed to be unidirectional with the fibres perpendicular to the crack faces. Tensile loads are considered to be applied in the direction of fibre alignment and the relationship between the strain applied to the composite and the process of crack growth is established.

The composite is assumed to consist of an intimate mixture of carbon fibres and glass fibres in a polymeric matrix. Two conditions are examined. Firstly it is assumed that the glass fibres are subject to chemical attack so that they fracture in the plane of the crack at low composite strain values. The crack is then considered to be bridged by the carbon fibres and the effect on crack stability of crack length, the carbon fibre polymer matrix interfacial debonding energy

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and the residual fractional shear strength of the debonded interface are examined.

The second condition considered is that in which chemical attack does not take place so that the glass fibres now have a higher failing strain that the carbon fibres. Matrix cracks are assumed to be produced as the carbon fibres fracture and to be bridged by the glass fibres. It is assumed that the carbon fibres will not be distributed uniformly and the analysis predicts that fracture initiates in regions of high carbon fibre concentration and propagates into regions of lower carbon fibre concentration. The effect of this process on the stability of the cracks is examined. Comparisons are made with the experimental observations, made by Aveston and Sillwood [8], of the behaviour under tensile loading of carbon fibre/glass fibre polymeric composites.

2. The analytical model

The analytical model used in this investigation has been described in detail elsewhere [3] and is given in outline here. The redistribution of strain due to the presence of a matrix crack is assumed to be confined within an elliptical zone around the crack having the crack as its minor axis and with its major axis three times the crack length (Fig. 1). In the absence of the crack bridging fibres the tensile strain carried by the matrix is assumed to increase linearly from zero at the crack face to the bulk strain carried by the composite, ε_β, at the edge of the elliptical zone. The zone is considered to be divided into a number of parallel strips. The strain energy carried by the material within the zone is then calculated by summing the strain energy carried by all of the strips. Each strip is assumed to develop strains only in the direction of loading. The rate of release of strain energy with increasing crack length is obtained by numerical differentiation and, when the reinforcing fibres are not present, this has the same numerical value as that obtained by Griffith [2] for a crack in an isotropic elastic solid using elasticity theory.

The strain field is modified by the presence of the reinforcing fibres and this effect is illustrated in Fig. 2, which shows the strain developed in a segment of the strain field parallel with the direction of the fibre alignment. The strain values are plotted on the vertical axis and the reinforcing fibres carry their maximum strain at the crack face. Equations describing any section of the strain field around the crack can be derived [3].

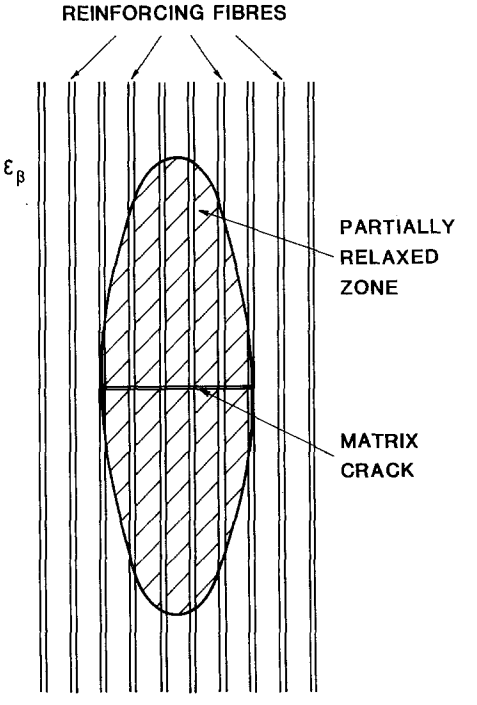

Figure 1 Illustrating the dimensions of the elliptical zone around the crack within which the strain field is disturbed by the crack.

$$\varepsilon_r = \{\varepsilon_\beta L_3/[Q(P + \varepsilon_\beta/L_3)^{-2} + L_3/\varepsilon_\beta]\}^{1/2},$$

$$\varepsilon_\mu = L_1(P + Q + \varepsilon_\beta/L_3),$$

$$L_1 = \varepsilon_r(P + \varepsilon_\beta/L_3)^{-1},$$

$$L_2 = L_3\,\varepsilon_r/\varepsilon_\beta,\ L_3 = 3(a^2 - z^2)^{1/2} \quad (1)$$

where $P = 2V_f\,\tau/E_m V_m r$, $Q = 2\tau/E_f r$, ε_r, ε_μ, ε_β, L_1, L_2, L_3 are defined in Fig. 2 and z is the distance from the centre of the crack to the position of the section considered.

The strain energy released by a parallel sided strip of width δz and unit thickness positioned at a distance z from the centre of the crack is given by δW_{Rz} where

$$\delta W_{Rz} = [E_c\,\varepsilon_\beta^2 L_3/2 - E_c\varepsilon_\beta^2(L_3^3 - L_2^3)/6L_3^2 - E_c\,\varepsilon_r^2(L_2 - L_1)/2 - V_m E_m \varepsilon_r^2 L_1/6 - V_f E_f(\varepsilon_\mu^2 + \varepsilon_\mu\varepsilon_r + \varepsilon_r^2)\,L_1/6]\,\delta z \quad (2)$$

E_f and E_m are the Young's modulus values of the fibres and matrix, respectively, and the other symbols have their usual meanings. Over the distance 0 to L_1 in Fig. 2 energy is absorbed in

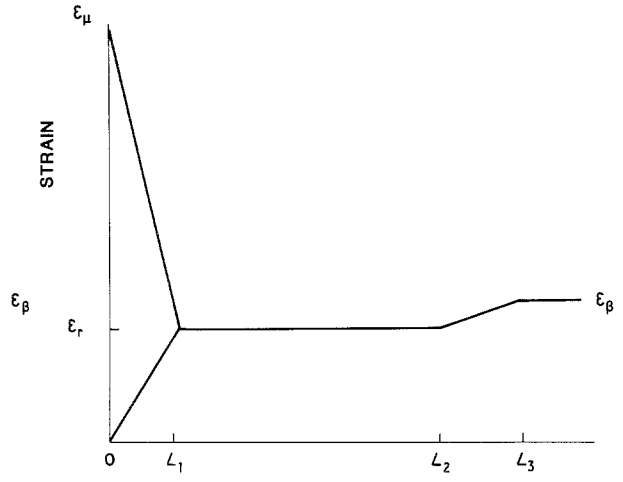

Figure 2 The assumed strain distribution along a strip of material aligned parallel to the fibres within a quadrant of the strain field illustrated in Fig. 1. Strain values plotted on the vertical axis. Distance from the crack face given by the horizontal axis.

rupturing the chemical bonding of the fibre matrix interface. If the energy required to debond the interface is G_d then the work done on one side of the crack per unit area of crack face will be given by $2V_f L_1 G_d/r$. Also, over the distance $0L_1$ differential movement between the fibres and matrix occurs so that energy is absorbed frictionally. The work done on one side of the crack per unit area of crack face can be show to be given by $V_f \tau \varepsilon_\mu L_1^2/3r$. During crack extension, energy is absorbed in rupturing the matrix and in fracturing some of the crack bridging fibres since a distribution of fibre strengths is assumed. If the crack is assumed to extend by a small amount, so that the proportion of fibres, N, which have fractured remains constant, the work of fracture of the matrix will be given by G_m where,

$$G_m = V_p W_{pf} + N V_{ft} W_{ff} \quad (3)$$

Here V_p is the volume fraction of the polymeric matrix, W_{pf} the work of fracture of the polymeric material, V_{ft} the total number of fibres present and W_{ff} the work of fracture of the fibres.

The proportion of the fibres which have fractured can be calculated from their known failing strain distributions and the value of the peak fibre strain, ε_μ, over the length of the crack. This can be calculated segment by segment or, more approximately, by taking an average value of ε_μ over the total length of the crack. In principle, any form of fibre failing strain distribution can be applied. In the case of the calculations performed here a rectangular distribution is assumed, the fibre failing strains being taken as distributed uniformly between a maximum and minimum value. If these are given respectively by ε_{max} and ε_{min} we have,

$$N = (\varepsilon_\mu - \varepsilon_{min})/(\varepsilon_{max} - \varepsilon_{min}) \quad (4)$$

The elastic modulus of the matrix measured in the direction of fibre alignment is assumed to be given by a simple rule of mixtures relationship. For the first case considered, in which all of the glass fibres are assumed to have fractured, the elastic modulus of the matrix is given by,

$$E_m = (V_p E_p + V_{GF} E_{GF} + N V_{ftc} E_{fc})/V_m \quad (5)$$

where V_{GF} and E_{GF} are respectively the volume fraction and Young's modulus of the glass fibres, and V_{ftc} and E_{fc} are respectively the total volume fraction and Young's modulus of the carbon fibres. Thus the proportion of the carbon fibres which will have fractured is given by NV_{ftc}. The volume fraction of the matrix is given by $(1 - V_f)$ where V_f is the number of fibres intact so that $V_f = (1 - N)V_{ftc}$.

In the second case examined all of the glass fibres are considered to be intact since ε_{max} for the carbon fibre is less than ε_{min} for the glass fibres. The matrix therefore consists of polymeric material plus carbon fibres. In the case of the systems considered the carbon fibres are present in low volume fractions so that they will be distributed non-uniformly. Crack stability is therefore considered for a range of carbon fibre volume fractions and crack sizes and the elastic modulus of the matrix is now given by,

$$E_m = (V_p E_p + V_{ftc} E_{fc})/V_m \quad (6)$$

and the work of fracture is given by,

$$G_m = (V_p W_{pf} + V_{ftc} W_{ffc}) \quad (7)$$

where V_{fc} is the volume fraction of carbon fibres present and W_{ffc} their work of fracture.

From Equation 2 the strain energy released by a parallel strip of material within the elliptical zone around the crack can be computed after inserting the appropriate values for the physical constants. The strain energy released by the whole elliptical zone can then be computed by numerical integration and the rate of release of strain energy with increasing crack length obtained by numerical differentiation. The stability of the crack can be established by comparing the amount of energy released with the amount of energy absorbed for a corresponding small increase in crack length. If the crack is stable (amount released less than amount absorbed) the bulk strain ε_β carried by the composite can be increased incrementally until the crack is predicted to be unstable. Under some circumstances the matrix crack may extend without causing fibres to fracture. Alternatively the strain carried by the crack bridging fibres may be sufficiently great to cause a proportion of them to fracture. If those remaining are insufficient to maintain the stability of the crack the model predicts that it will extend catastrophically by the sequential failure of the remaining crack bridging fibres.

3. Analysis of the growth of cracks in a glass fibre/carbon fibre intermingled hybrid by stress corrosion processes

Here we assume that the strength of the glass fibres can be taken as negligible and that they fail in the plane of the crack. The model predicts the observed localized failure of the glass fibres under these conditions because of the enhanced stresses carried in the plane of the crack by all crack bridging fibres.

The system considered was assumed to comprise the materials which are listed in Table I.

TABLE I

Carbon fibres	Volume fraction 0.05 Young's modulus 200 GPa, diameter 10 μm $\varepsilon_{max} = 0.018\,75$ $\varepsilon_{min} = 0.013\,75$
Glass fibres	Volume fraction 0.4275 Young's modulus 72 GPa
Polymeric matrix	Volume fraction 0.5225 Young's modulus 3 GPa

The Young's modulus of the glass fibres plus the polymeric matrix is thus 34 GPa and this is regarded as being reinforced by a volume fraction of 0.05 of carbon fibres. An examination has been made of the stability of cracks having a total length of 0.002 and 0.02 m. The effect of varying the chemical bonding between the carbon fibres and the matrix over a wide range has been considered. The influence of two levels of residual frictional interaction (1 and 10 MPa) between the carbon fibres and the polymeric matrix after interfacial debonding have also been studied.

The conditions for crack stability and mode of failure were computed in the following way. The half crack length was taken as either 0.001 or 0.01 m and the residual frictional interfacial interaction as 1 or 10 MN m^{-2}. An arbitrary value of the debonding energy G_D was then taken. The crack stability was then calculated for successively decreasing values of N (the proportion of crack bridging carbon fibres assumed to have fractured). The elastic modulus of the matrix was computed for each value of N from Equation 5. These calculations gave the critical composite values $\varepsilon_{\beta c}$ at which the matrix crack would propagate. As the values of N were decreased the proportion of intact crack bridging fibres correspondingly increased so that the values of $\varepsilon_{\beta c}$ increased also. At each value of $\varepsilon_{\beta c}$, corresponding to successive decreases in the value of N, the proportion of crack bridging fibres which would have been expected to fracture, N^1, was computed from the maximum strains carried by the crack bridging fibres, ε_μ, and the assumed distribution of fibre failing strains. The computed value of N^1 increases as N decreases due to the progressive increase in $\varepsilon_{\beta c}$. The critical condition for catastrophic failure by the sequential failure of the crack bridging fibres is reached for the value of $\varepsilon_{\beta c}$ at which $N = N^1$ since $N^1 > N$ for higher values of ε_β.

For small values of G_D the value of N^1 may remain smaller than N, i.e. negative, when N is zero. The propagation of the crack is not now initiated by the failure of the crack bridging fibres although these may fail eventually as the crack length increases at a constant composite strain value.

The effects of various composite characteristics on crack stability computed as indicated above are illustrated in Figs. 3 and 4. Fig. 3

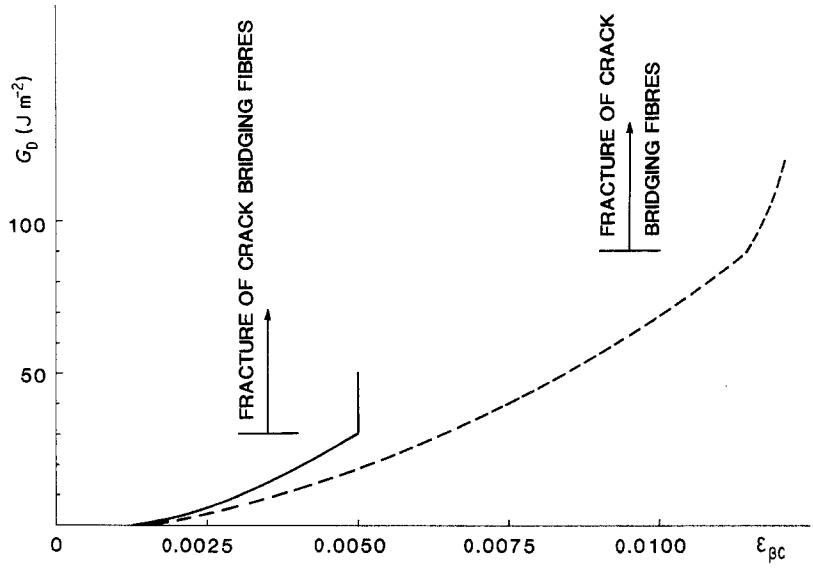

Figure 3 Illustrating the effect of the fibre/matrix debonding energy, G_D, on the critical strain for crack extension, $\varepsilon_{\beta c}$. Residual frictional interfacial shear strength after debonding, $\tau = 1\,\mathrm{MN\,m^{-2}}$. —— denotes half the crack length of 0.01 m, – – – denotes half crack length of 0.001 m.

refers to τ values of $1\,\mathrm{MN\,m^{-2}}$ and half crack lengths, a, of 0.001 and 0.01 m. When the interfacial debonding energy, G_D, is $50\,\mathrm{J\,m^{-2}}$ the analysis indicates that a crack of half length 0.001 m will propagate at a composite strain of 0.0084 without an immediate increase in the proportion of fractured crack bridging fibres. On the other hand, a crack of half length 0.01 m will propogate at a composite strain, $\varepsilon_{\beta c}$, of 0.05 by the sequential failure of the crack bridging fibres at the same fibre matrix debonding energy of $50\,\mathrm{J\,m^{-2}}$. For interfacial debonding energies in excess of $30\,\mathrm{J\,m^{-2}}$ the analysis suggests that cracks of half lengths of 0.01 m will propagate by the failure of the crack bridging fibres. If the crack length is diminished to 0.001 m this critical condition is not encountered until the interfacial debonding energy G_D reaches a value of $90\,\mathrm{J\,m^{-2}}$. The effect of increasing the residual frictional interfacial shear strength to $10\,\mathrm{MN\,m^{-2}}$ after debonding is indicated in Fig. 4. The analysis thus provides a means whereby the critical strain for the growth of a crack of arbitrary length can be computed and in

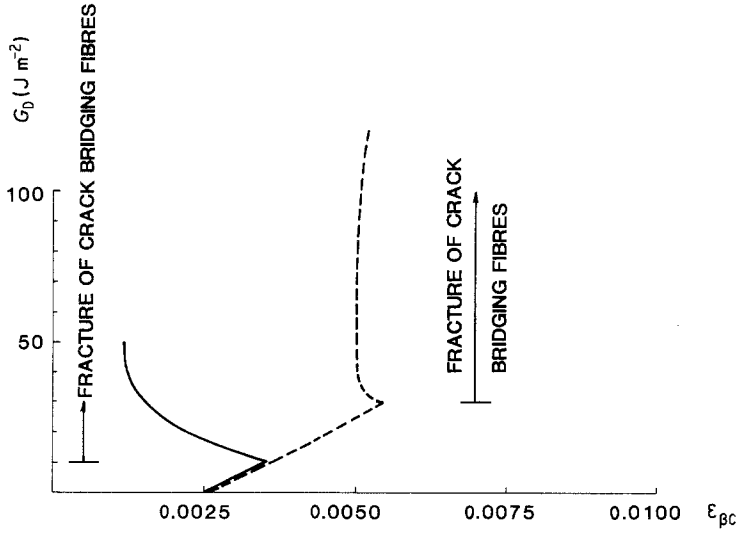

Figure 4 Illustrating the effect of the fibre/matrix debonding energy on the critical strain for crack extension, $\varepsilon_{\beta c}$. Residual frictional interfacial shear strength after debonding, $\tau = 10\,\mathrm{NM\,m^{-2}}$. —— denotes half crack length of 0.01 m, – – – denotes half crack length of 0.001 m.

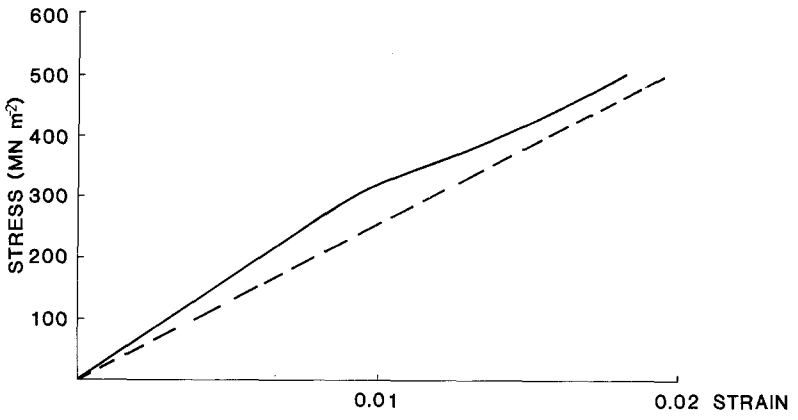

Figure 5 Stress–strain diagram for carbon fibre/glass fibre epoxy resin hybrid composite. Redrawn from Aveston and Sillwoood [8]. Theoretical stress–strain curve for glass fibre plus resin indicated thus ——.

addition, as the crack increases in length, the critical crack length at which failure occurs by the sequential failure of the intact crack bridging fibres can be obtained. For the glass fibre, carbon fibre polymer matrix system considered here, the proportion of carbon fibres which would be required to maintain the stability of a matrix crack of arbitrary length in a corrosive environment at an arbitrary composite strain value can be estimated.

3.1. Discussion

The computations illustrated in Figs. 3 and 4 indicate that the addition of a relatively small volume fraction (0.05) of carbon fibres would be expected to suppress the growth of a crack in a hybrid carbon glass system when the glass fibres fracture at low strain values due to a stress corrosion mechanism. As would be expected, the computed stress for unstable crack growth increases as the crack length decreases. The calculations also indicate that the process of failure will be influenced by the degree of chemical bonding of the fibre matrix interface and by the residual frictional shear strength of the debonded interface.

Two regimes for crack growth are possible. If the value of G_D is small a matrix crack encompassing the polymeric matrix and the glass fibres will grow without causing immediate failure of the carbon fibres. At high values of G_D the growth of the crack is accompanied by the sequential failure of the crack bridging fibres. The analysis given here assumes immediate failure of the glass fibres by chemical attack. In a practical situation this process will take time, the duration of which will be influenced by the stresses carried by the glass fibres since they will bridge the crack prior to failure. Thus, when the carbon fibres remain intact, the matrix crack would be expected to grow slowly because of the finite time required for the chemical degradation of the glass fibres. Catrastrophic failure would not be expected to occur until the crack length is large enough to cause sequential failure of the crack bridging carbon fibres at the applied strain values.

4. Failure of glass fibre/carbon fibre hybrid in the absence of stress corrosion effects

Glass fibre/carbon fibre unidirectionally reinforced polymer bonded hybrid composites have been studied by Aveston and Sillwood [8]. They observed that the load–extension curve for a hybrid loaded in tension showed a marked inflection at a strain of about 0.01 (Fig. 5). At strains below the point of inflection the slope of the stress–strain curve corresponded approximately to the rule of mixtures value. Above the point of inflection the slope of the load–extension curve corresponded to that of the glass fibres plus polymeric material. Thus at strains below the point of inflection the carbon fibres carried an increasing load with increasing strain and above the point of inflection a constant load independent of strain. The inflection occurs at a strain value about twice that of the normal failing strain of the fibres and the acoustic emission from the specimens had a maximum value in this region.

Aveston and Sillwood pointed out that these

results were compatible with two possible crack inhibiting mechanisms. Firstly the glass fibres and the polymeric material could be regarded as a high elongation matrix reinforced by low elongation fibres. It can be argued on energetic grounds that the presence of the intact matrix will inhibit the fracture of the lower failing strain fibres so that they fracture at higher strain values. Aveston and Sillwood examined this argument and showed that, for a simple frictionally bonded system the enhanced failing strain of the fibres ε_{fuc} is given by,

$$\varepsilon_{fuc}^2 = \frac{6\gamma_f V_m^2 E_m}{x^1 E_c E_f V_f} \quad (8)$$

where γ_f is the surface energy of the fibre fracture surfaces, E_c is the hybrid composite Young's modulus. E_f and V_f and E_m and V_m are, respectively, the Young's modulus and volume fraction of fibres and matrix. The value of x^1 is given by the fibre stress transfer length, i.e. the distance from the point of fracture required for the stress on the fibre to build up to its original value. Equation 8 implies a very high enhancement in the brittle fibre failure strains as the fibre volume fraction tends to zero.

Aveston and Sillwood observed a regular pattern of white striations in the composite after it was strained above the inflection point although no cracks as such could be observed. The striations had a spacing of about 1 mm and were considered to correspond to debonded interfaces at breaks in the carbon fibres. By taking their spacing to be governed by the stress transfer distance and to lie between x^1 and $2x^1$ the enhanced fibre breaking strain value ε_{fuc} was deduced by Aveston and Sillwood from Equation 8.

The second mechanism suggested by Aveston and Sillwood to account for the form of their experiment stress–strain curves was that cracks emanating from individual carbon fibres or carbon fibre bundles were prevented from propagating by the presence of the intact glass fibres. This hypothesis is considered in more detail here using the model developed in Section 3 of this paper suitably modified.

The matrix is now considered to be formed from the carbon fibres plus the polymeric adhesive, both of which fracture at the normal carbon fibre failing strain. This "matrix" is reinforced by the intact glass fibres which inhibt the growth of cracks in the polymer bonded carbon fibre material. Because of the mode of fabrication used by Aveston and Sillwood considerable local variations in carbon fibre content would be expected. Hence the local elastic modulus of the "matrix" would be expected to vary over a considerable range of values.

Calculations of the conditions for crack instability were made for a composite strain of 0.01, corresponding to the inflection in Fig. 5. Two crack lengths, 0.002 and 0.02 m, were considered and the relationship between the chemical debonding energy of the glass fibre polymer matrix interface, G_D, and the residual frictional shear strength of the interface after debonding, τ, on the stability of the cracks was computed. The curves shown in Figs. 6 to 8 indicate the limits of matrix crack stability for the various carbon fibre volume fractions shown in Table II, using the theoretical analysis given in Section 3. Each curve relates to a particular volume fraction of carbon fibres and the crack will be stable for debonding energy values, G_D, equal to or greater than the values denoted by the curves. The Young's modulus of the carbon fibres was taken as 300 GPa to correspond with the data given by Aveston and Sillwood. The work of fracture of the matrix was computed from the relative proportions of carbon fibres and polymeric material from which it was formed. The work of fracture of the polymeric material was taken as 200 J m^{-2} and that of the carbon fibres

TABLE II Matrix characteristics used in the computation of the data shown in Figs. 6, 7 and 8

Carbon fibre volume fraction	Volume fraction of polymer	Glass fibre volume fraction	Young's modulus of matrix (GN m^{-2})	Work of fracture of matrix (J m^{-2})
0.035	0.615	0.35	19.0	197
0.10	0.55	0.35	48.7	192
0.20	0.45	0.35	94.4	185
0.30	0.35	0.35	140.0	177
0.40	0.25	0.35	186.0	169
0.50	0.15	0.35	231.0	162

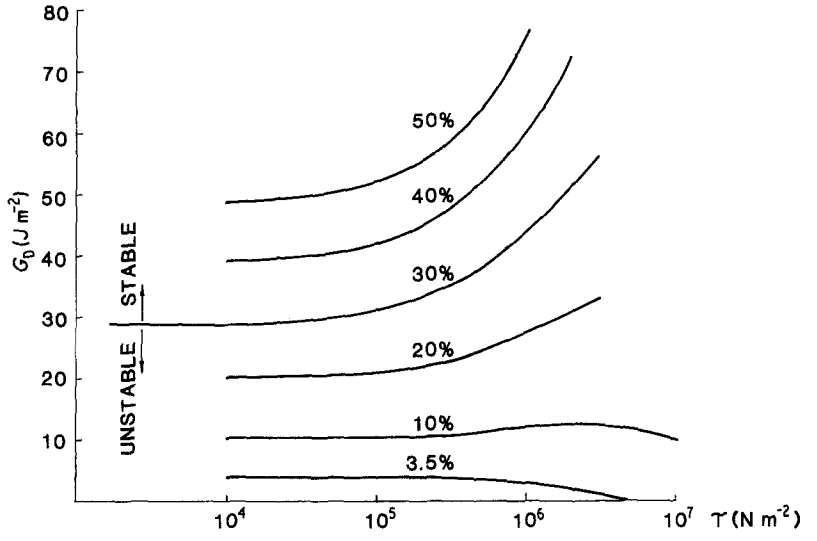

Figure 6 Computed values of the debonding energies, G_D, and the corresponding residual frictional interfacial shear strength values, τ, at which a matrix crack of total length 0.002 m is just stable at a composite strain of 0.01. The individual curves relate to various concentrations of carbon fibres in the "matrix". Stability conditions indicated.

was taken to be 150 J m^{-2}. In Figs. 6 and 7 the glass fibres were assumed to behave as individual filaments having a diameter of 13 μm. In Fig. 8 the glass fibres were assumed to be behaving as bundles of 204 filaments, the form in which the glass fibres strands were constructed. The bundles were assumed to have an effective diameter of 208 μm and an elastic modulus of 58 GPa. Computations were not carried out for crack lengths of 0.002 m when the reinforcement occurred by glass fibre bundles since the crack size would then be comparable with the fibre diameter and invalidate the physical basis of the analytical model.

From Figs. 6 and 7 it is apparent that, when the glass fibres behave as individual filaments, cracks of the lengths considered could remain stable over a very wide range of values of τ. Also, for realistic values of interfacial chemical debonding energies for the glass fibre polymer matrix interface, cracks would be expected to remain stable in regions having quite high local

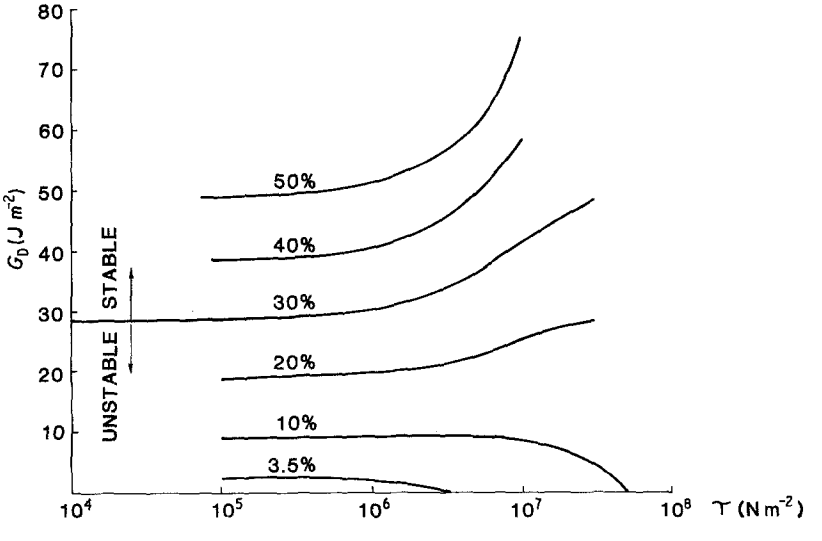

Figure 7 Computed values of the debonding energies, G_D, and the corresponding residual frictional interfacial shear strength values, τ, at which a matrix crack of total length 0.0002 m is just stable at a composite strain of 0.01. The individual curves relate to various concentrations of carbon fibres in the "matrix". Stability conditions indicated.

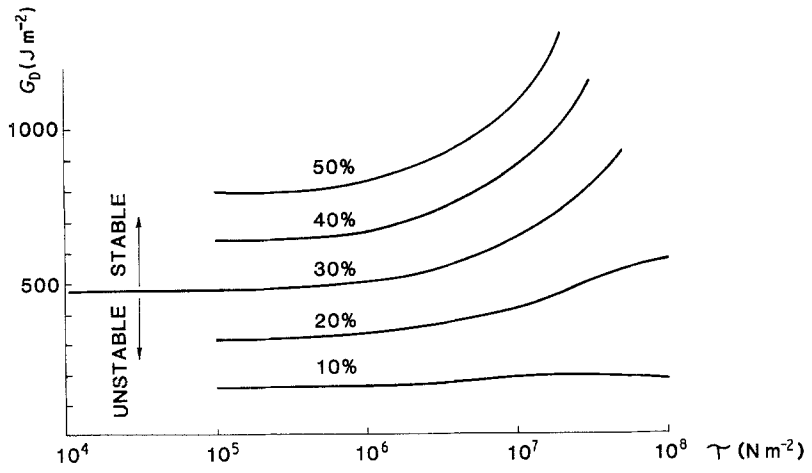

Figure 8 Computed values of the debonding energies, G_D, and the corresponding residual frictional interfacial shear strength values, τ, at which a matrix crack of total length 0.002 is just stable at a composite strain of 0.01. The individual curves relate to various concentrations of carbon fibres in the "matrix". Stability conditions indicated. Reinforcement assumed to be by glass fibre bundles having an effective diameter of 208 μm.

concentrations of carbon fibres. Furthermore, for residual frictional interfacial shear strengths, τ, of less than about 5 MPa the stability criteria are not strongly dependent on the lengths of the "matrix" cracks considered in the calculation (0.02 and 0.002 m).

For the range of adhesive bonding energies considered in the computation, the sequence of failure indicated by Figs. 6 and 7 can be assumed to be as follows. At the composite strain value studied (0.01) and for sufficiently low values of G_D, cracks will form in local regions of high carbon fibre concentration. In propagating these cracks will, of necessity, extend into regions of lower carbon fibre concentration. Thus, the crack, although increased in size, may become stable since the average carbon fibre concentration over the total area of the crack is reduced and crack stability can be maintained with a correspondingly reduced debonding energy value G_D. These conclusions are, of course, only qualitative and the calculations would have to be performed over a wide range of conditions, e.g. strain values and crack sizes, in order to arrive at comprehensive numerical estimates. Nonetheless, the predictions of the analysis are consistent with the experimental data obtained by Aveston and Sillwood if it is assumed that the inflection in their load–extension curve (Fig. 5) at a strain of about 0.01 corresponds to the point at which large numbers of initially stable microcracks existing around local regions of high carbon fibre concentration start to extend and to multiply. This process will cease when the carbon fibres have fractured into lengths ranging from about x' to $2x'$ where $2x'$ is the critical fibre length. The slope of the load–extension curve at higher strain values would then be governed by the elastic properties of the glass fibres and the polymeric adhesive which is observed. The short lengths of carbon fibre would support a constant load at higher strains with a magnitude approximately that indicated by the offset of the load–extension curve shown in Fig. 5. Also, since frictional displacements would occur over the carbon fibre/polymer interface some degree of residual strain would be expected after unloading and this was also observed by Aveston and Sillwood.

The computations described in Figs. 6 and 7 are repeated in Fig. 8 where it is assumed that the glass fibres are behaving as 204 filament bundles. A similar relationship to that illustrated in Figs. 6 and 7 is obtained but the magnitude of the interfacial debonding energies required to maintain the stability of the "matrix" cracks is very much greater, and does not seem likely to be physically achievable except in regions of relatively low carbon fibre concentration.

Over the range of conditions studied the maximum strains carried by the crack bridging glass fibres were lower than those assumed necessary to initiate fracture (0.02). Thus the model predicts that multiple matrix cracking will occur

at composite strains lower than those required to cause failure of the glass fibres.

4.1. Discussion

Aveston and Sillwood suggested that one possible explanation of the apparently enhanced breaking strain of the carbon fibres in the glass fibre carbon fibre hybrids which they studied could be due to the ability of intervening glass fibres to prevent cracks extending from a failed cluster of carbon fibres. The analysis put forward here supports this view, and also enables predictions to be made about the relative significance of various physical parameters on crack inhibition. Both the debonding energy of the glass fibre/polymer matrix interface, G_D, and the diameter of the glass fibres are shown by the analysis to be of considerable significance in inhibiting crack growth. On the other hand, the process is unaffected by very considerable changes in the residual frictional interaction with the matrix of the debonded glass fibres. It should, however, be borne in mind that these conclusions refer only to the particular hybrid system studied here and might well be modified in the case of other fibre combinations.

5. Conclusions

An analysis of the growth of cracks transversely to the fibres in a unidirectional hybrid glass fibre/carbon fibre system subjected to tensile strains has been put forward. Two circumstances have been considered: one in which the glass fibres fracture at low strains due to environmental corrosion and one in which these effects are absent so that the carbon fibres fracture while the glass fibres remain intact. The latter situation has previously been studied by Aveston and Sillwood [8]. The observations of crack suppression by Aveston and Sillwood are consistent with the analysis put forward here. The effect is predicted to be dependent on the fibre/matrix interfacial characteristics and on the fibre diameters and would be expected to occur at values typical of practical composites.

The behaviour of carbon fibre/glass fibre hybrid systems under stress corrosion has also been examined. Here it is considered that the glass fibres fail at negligible strain values and that the crack so formed is inhibited by the still intact carbon fibres. The analysis indicates that a relatively low volume fraction of carbon fibres will inhibit the growth of cracks in the glass fibres. This effect is predicted to be sensitive to the degree of chemical bonding and residual frictional coupling between the carbon fibres and the polymeric matrix.

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